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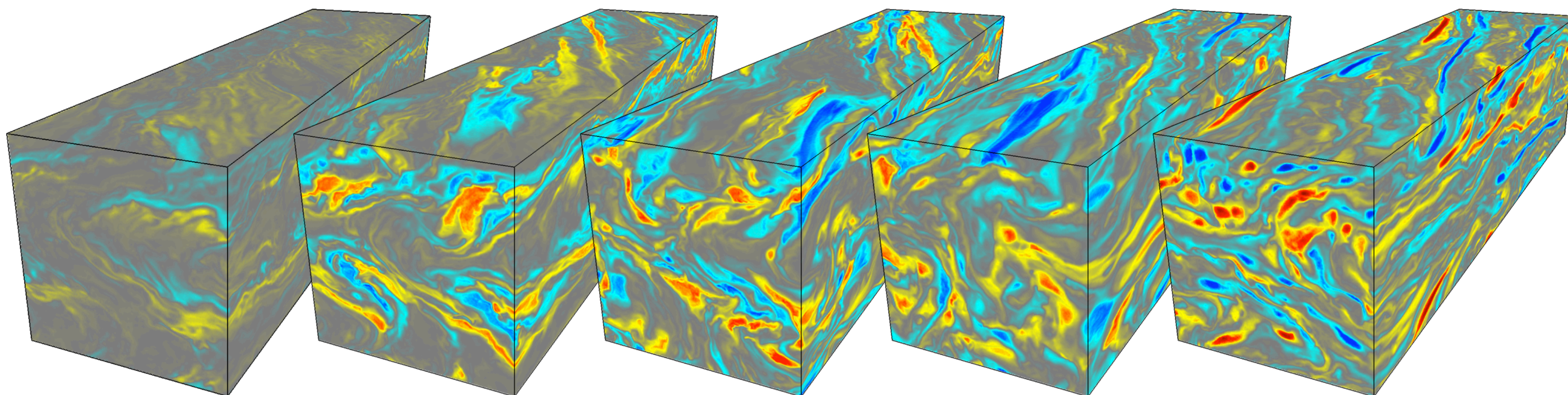
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# *Magnetorotational Turbulence and Dynamo in a Collisionless Plasma*



*Phys. Rev. Lett.* **117**, 235101(2016)



Low-luminosity black-hole accretion flows are weakly collisional

e.g., Galactic center Sgr A\*

at  $R \sim 0.1$  pc... ( $\sim 10^5 R_{\text{horizon}}$ )

$$n \sim 100 \text{ cm}^{-3} \quad \lambda_{\text{mfp}} \sim 0.01 \text{ pc}$$

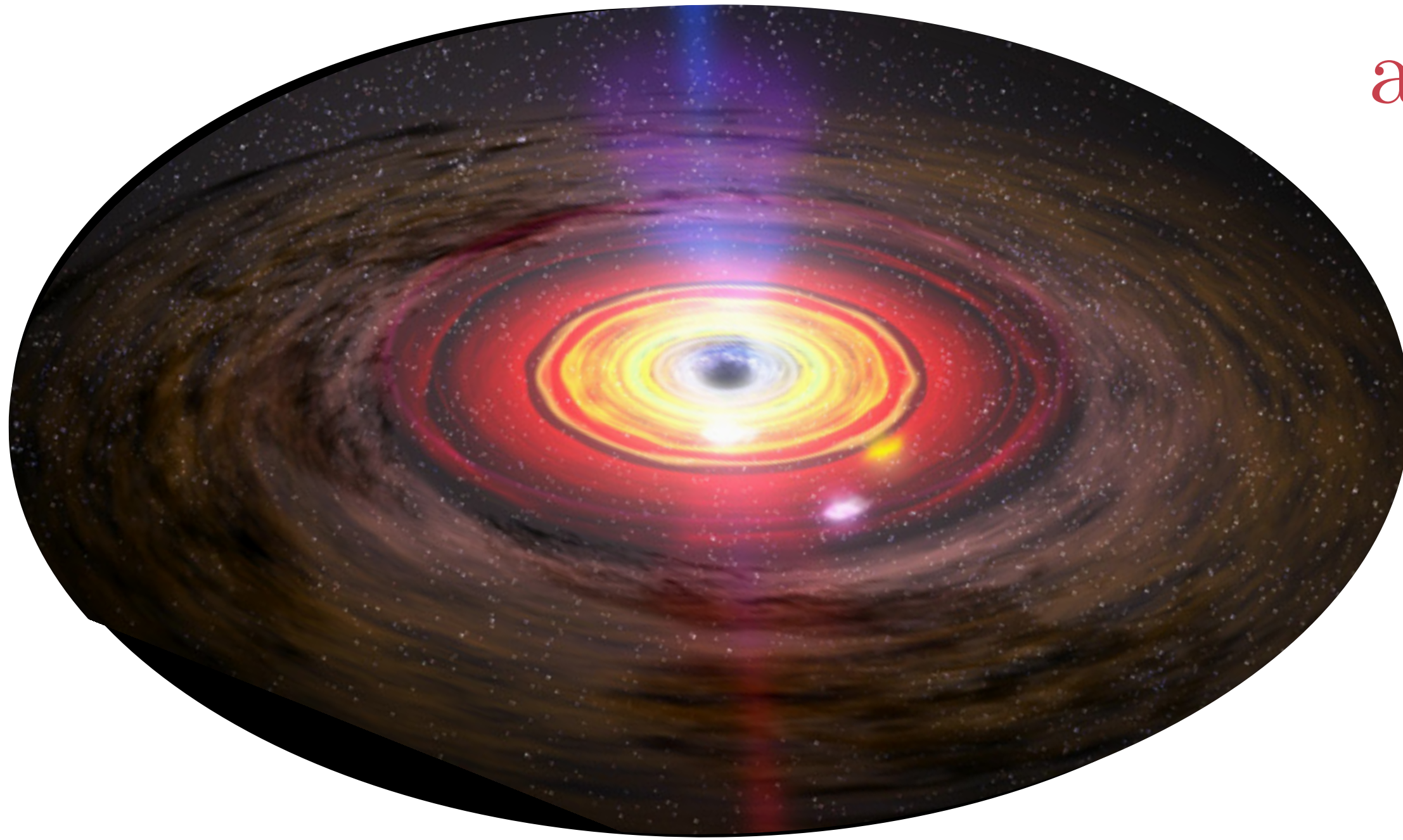
$$k_B T \sim 2 \text{ keV} \quad \rho_i \sim 1 \text{ ppc}$$

$$B \sim 1 \text{ mG} \quad \Omega_i \sim 10 \text{ s}^{-1}$$

becomes collisionless within this radius

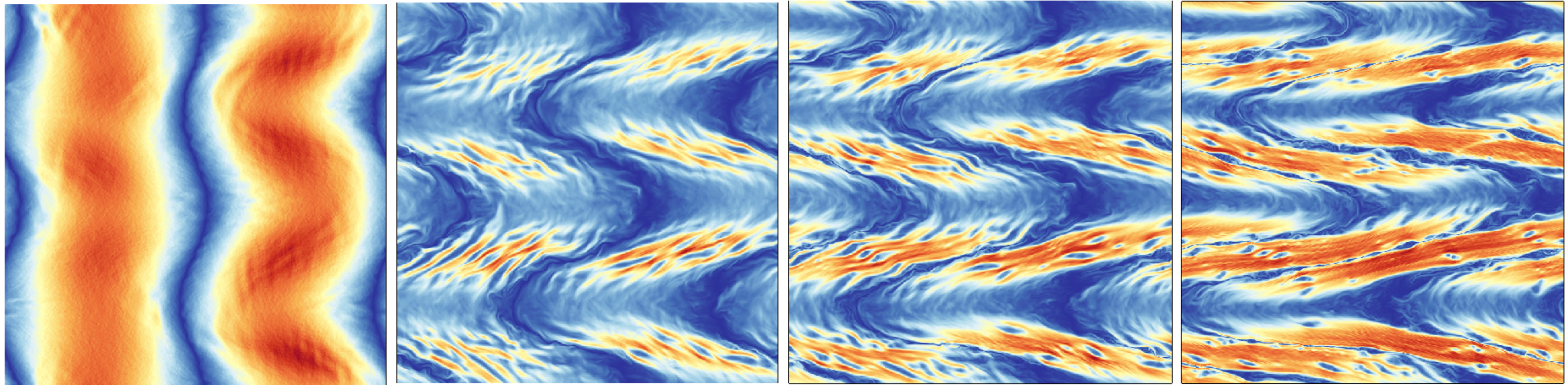
a **kinetic** approach is thus necessary to understand:

- transport of heat and angular momentum
- particle energization
- growth and structure of magnetic field





Magnetorotational Instability (MRI) amplifies magnetic fields and drives angular-momentum transport, even in collisionless plasmas



$|B|$

time →

demonstration of MRI “channel modes” in collisionless plasma

*new feature:* MRI drives pressure anisotropy, which triggers kinetic instabilities

*see Riquelme et al. 2012 & Hoshino 2013 for more on kinetic MRI in 2D*

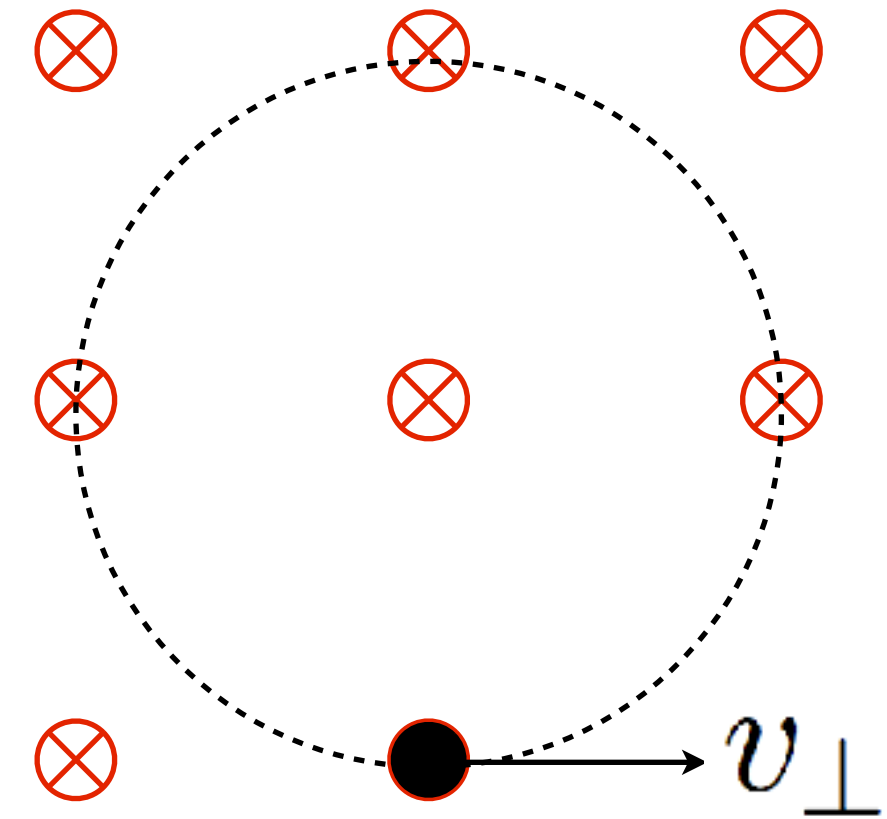
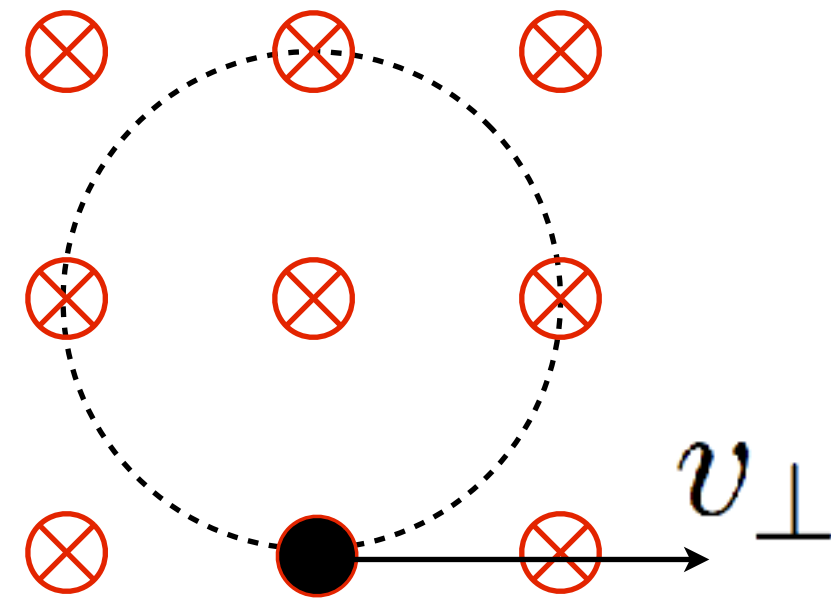


*Change magnetic-field strength, generate velocity-space anisotropy...*

adiabatic invariants:  $\frac{d}{dt} \oint \mathbf{p} \cdot d\mathbf{q} \simeq 0$

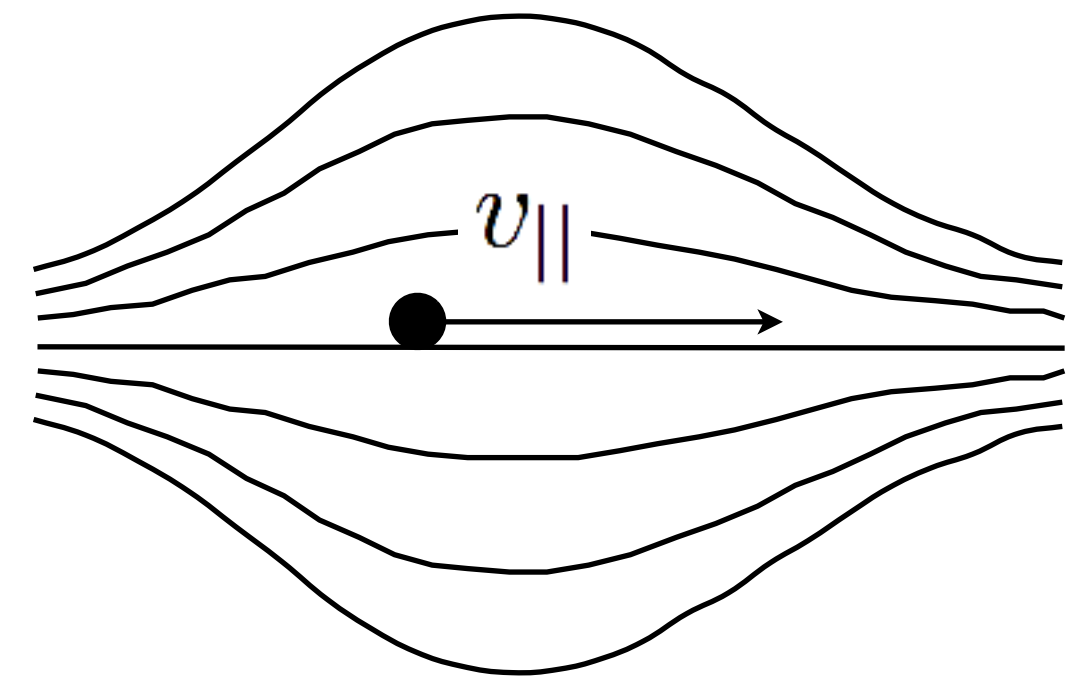
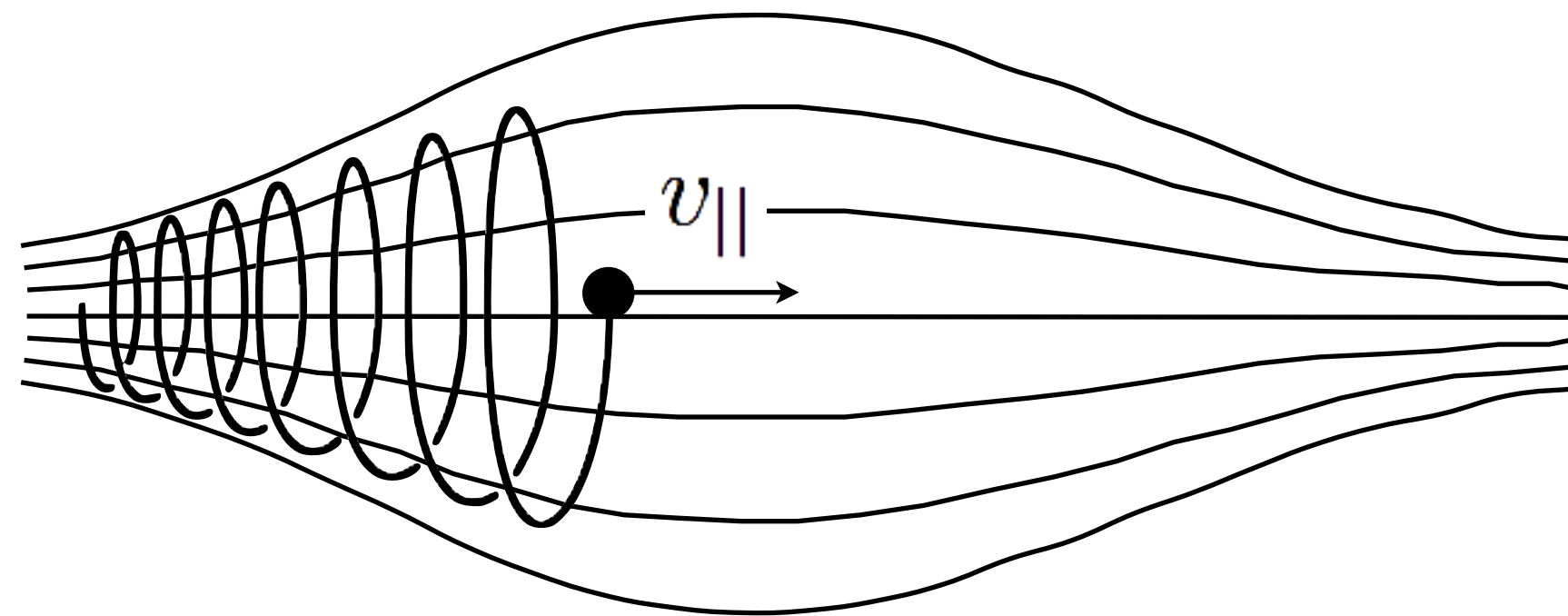
$$\mu = \frac{mv_{\perp}^2}{2B}$$

Kruskal (1958)



$$J = \oint d\ell_B m v_{\parallel}$$

Northrop & Teller (1960)



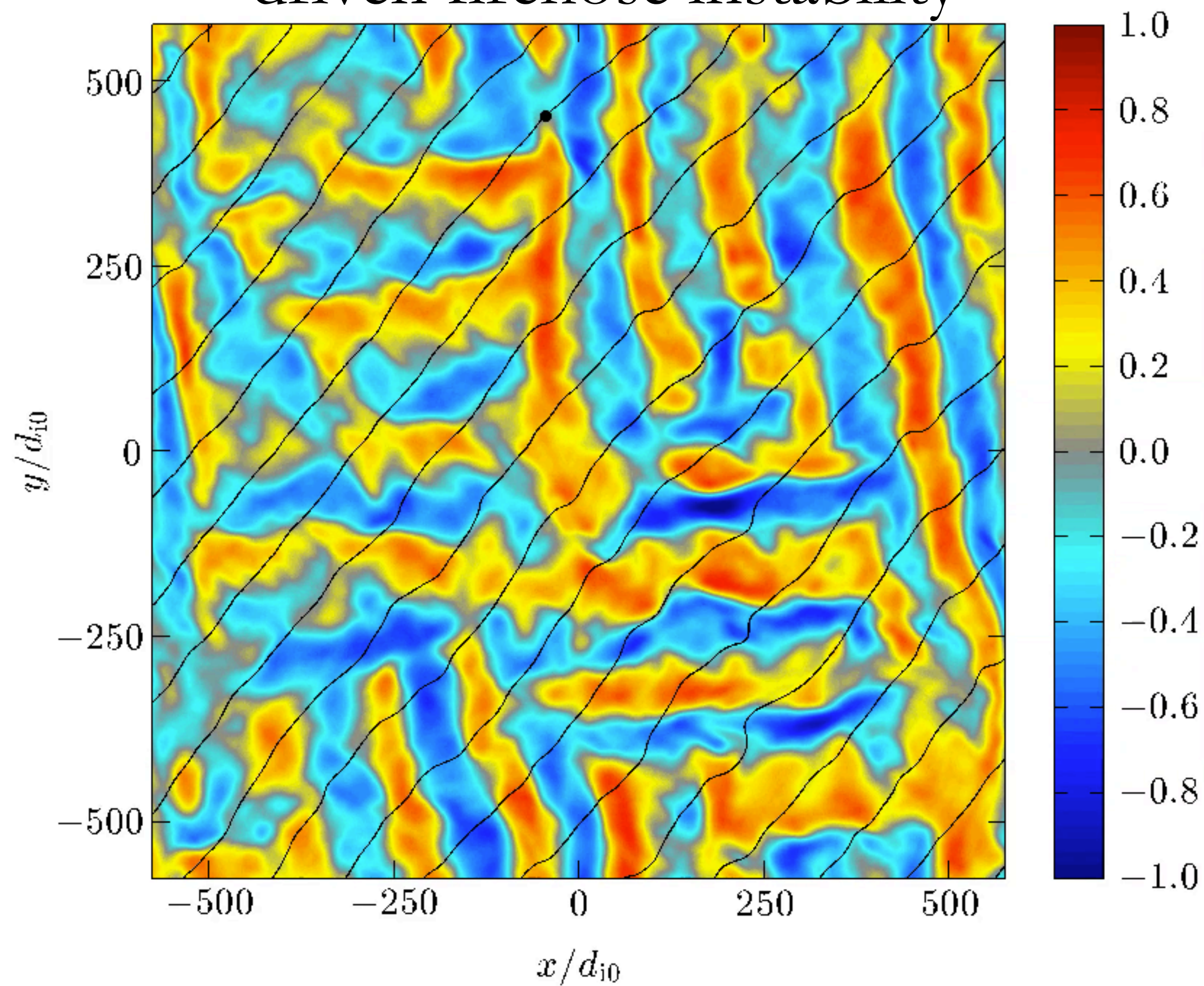


field-biased **pressure anisotropy** is important:

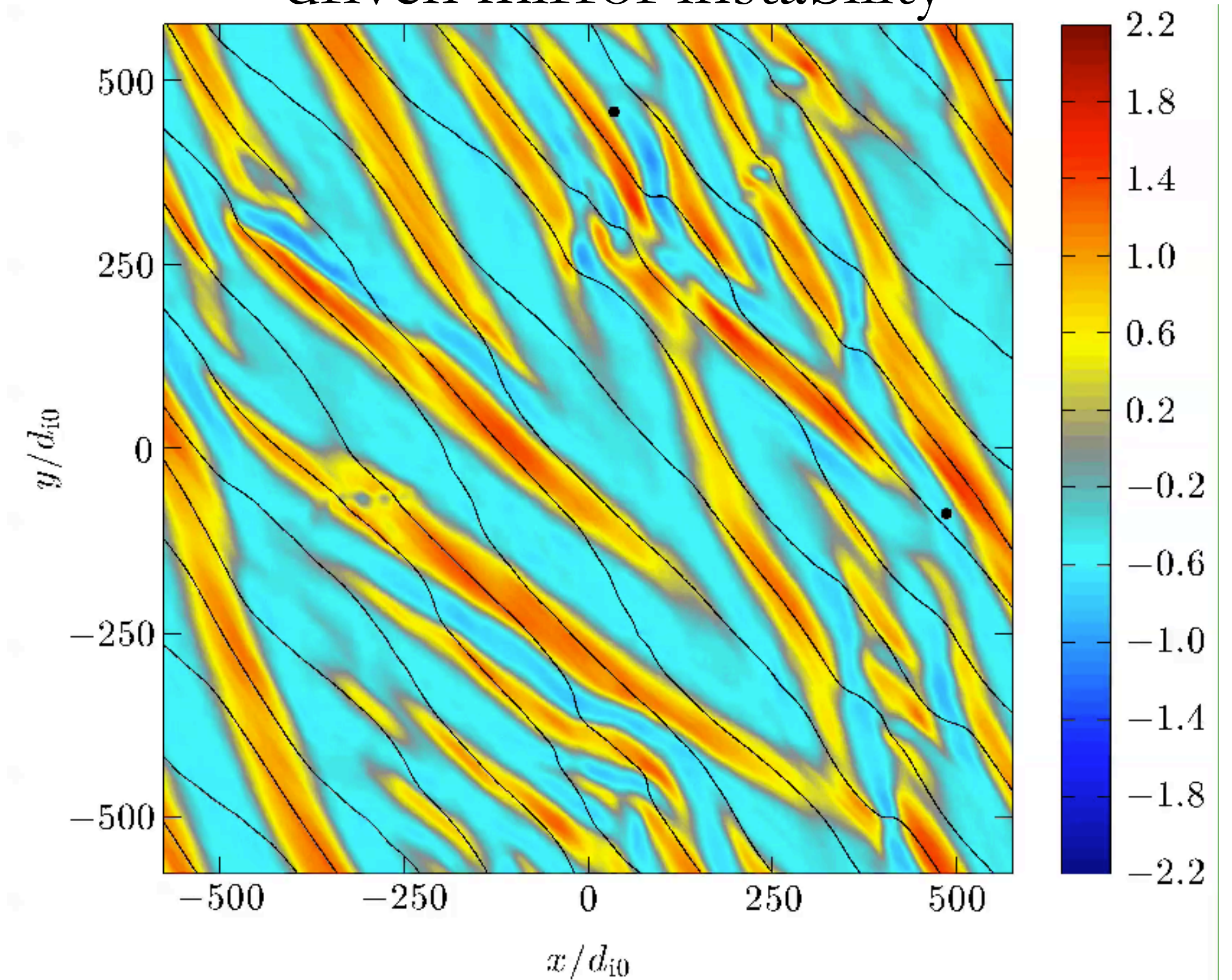
- triggers Larmor-scale instabilities, which set effective viscosity (and conductivity)

Kunz, Schekochihin & Stone (2014), *Phys. Rev. Lett.*

driven firehose instability



driven mirror instability





field-biased **pressure anisotropy** is important:

- modifies linear MRI (Quataert, Dorland & Hammett 2002), nonlinear MRI (Squire, Quataert & Kunz 2017), and MRI turbulence (Sharma et al. 2006)

stress transporting angular momentum:

$$T_{R\phi} = \rho u_R u_\phi - \frac{B_R B_\phi}{4\pi} \left( 1 + \frac{p_\perp - p_\parallel}{B^2/4\pi} \right)$$

Reynolds  
stress

Maxwell “viscous”  
stress stress



# hybrid-kinetic PIC shearing-box simulation of 3D3V magnetorotational turbulence and dynamo, using *Pegasus*

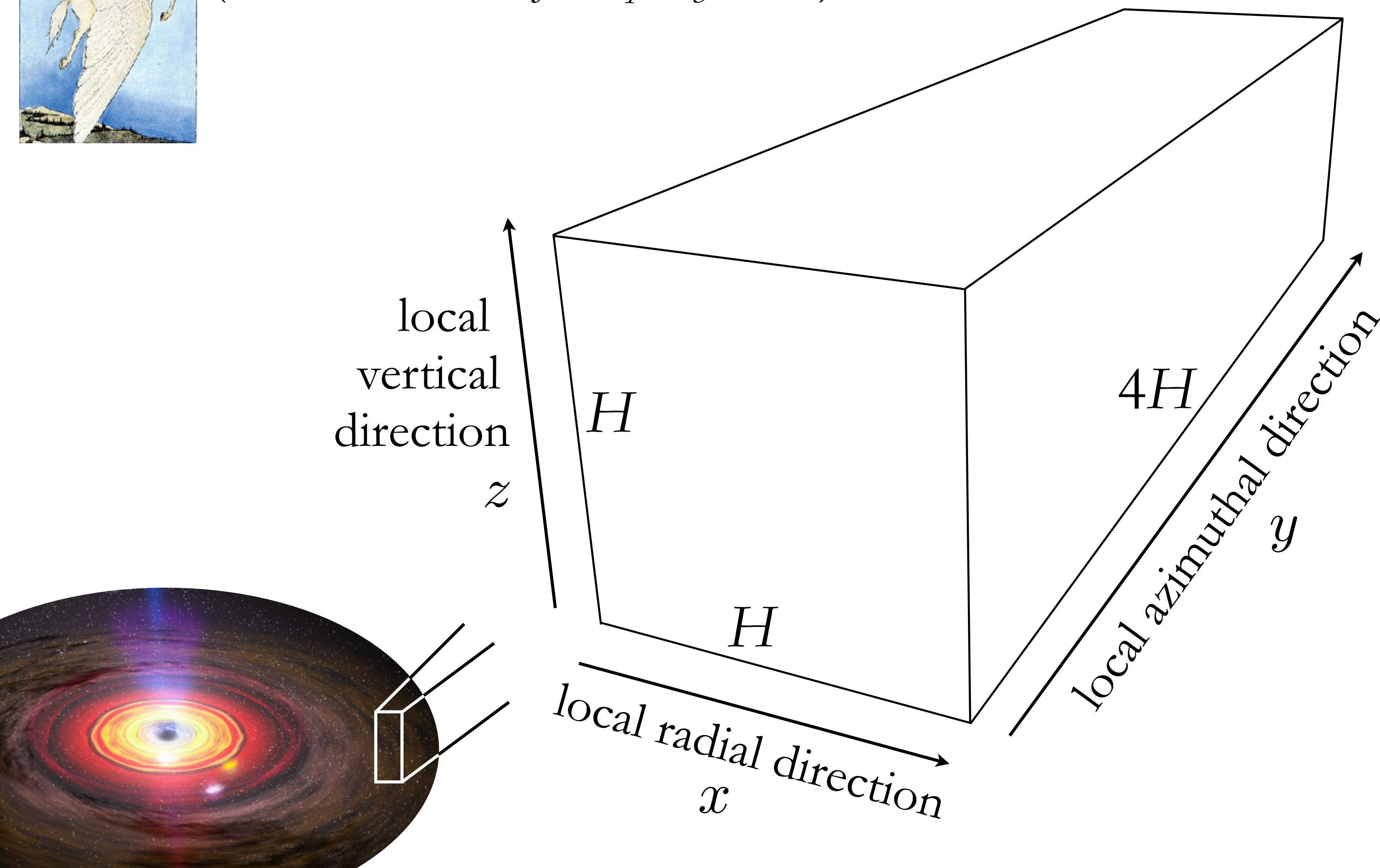
(Kunz, Stone & Bai, *J. Comp. Phys.* 2014)



$$H \doteq \frac{v_{\text{th}}}{\Omega_{\text{rot}}} = 50 \text{ initial ion Larmor radii}$$

$$\beta_0 = 400 \quad \text{zero-net flux}$$

$$\text{Rm} \doteq \Omega_{\text{rot}} H^2 / \eta = 37,500$$



384 x 1536 x 384 cells  
with 14.5 billion particles



$$\left(\frac{\partial}{\partial t}-\frac{3}{2}\Omega_{\mathrm{rot}}x\frac{\partial}{\partial y}\right)f+\boldsymbol{v}\cdot\boldsymbol{\nabla}f+\left[\frac{Ze}{m}\left(\boldsymbol{E}+\frac{\boldsymbol{v}}{c}\times\boldsymbol{B}\right)-2\Omega_{\mathrm{rot}}\hat{\boldsymbol{e}}_z\times\boldsymbol{v}+\frac{3}{2}\Omega_{\mathrm{rot}}v_x\hat{\boldsymbol{e}}_y\right]\cdot\frac{\partial f}{\partial\boldsymbol{v}}=0,$$

$$\boldsymbol{E}+\frac{\boldsymbol{u}}{c}\times\boldsymbol{B}=-\frac{T_e}{e}\boldsymbol{\nabla}\ln n+\frac{(\boldsymbol{\nabla}\times\boldsymbol{B})\times\boldsymbol{B}}{4\pi Zen}$$

$$\left(\frac{\partial}{\partial t}-\frac{3}{2}\Omega_{\mathrm{rot}}x\frac{\partial}{\partial y}\right)\boldsymbol{B}=-c\boldsymbol{\nabla}\times\boldsymbol{E}-\frac{3}{2}\Omega_{\mathrm{rot}}B_x\hat{\boldsymbol{e}}_y$$

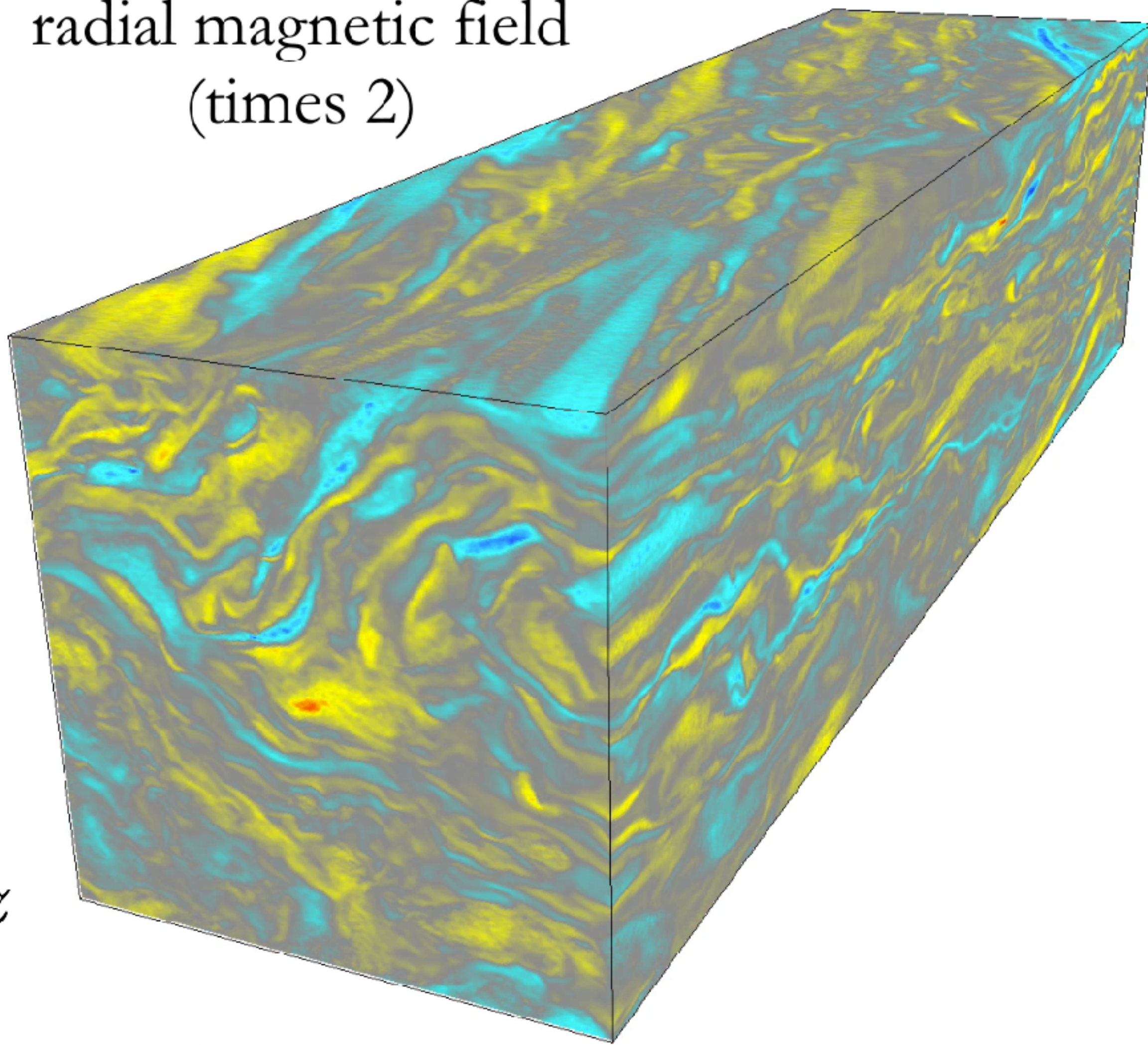


## Questions to address:

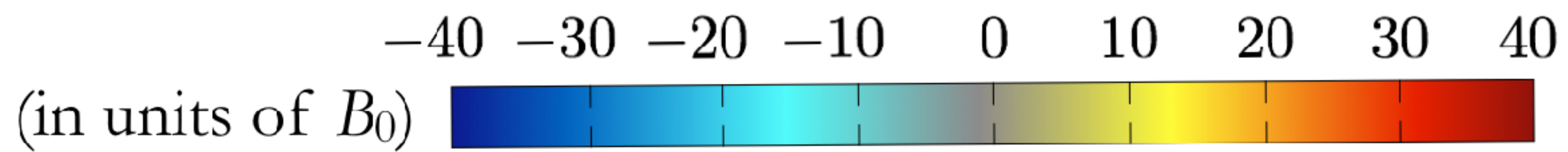
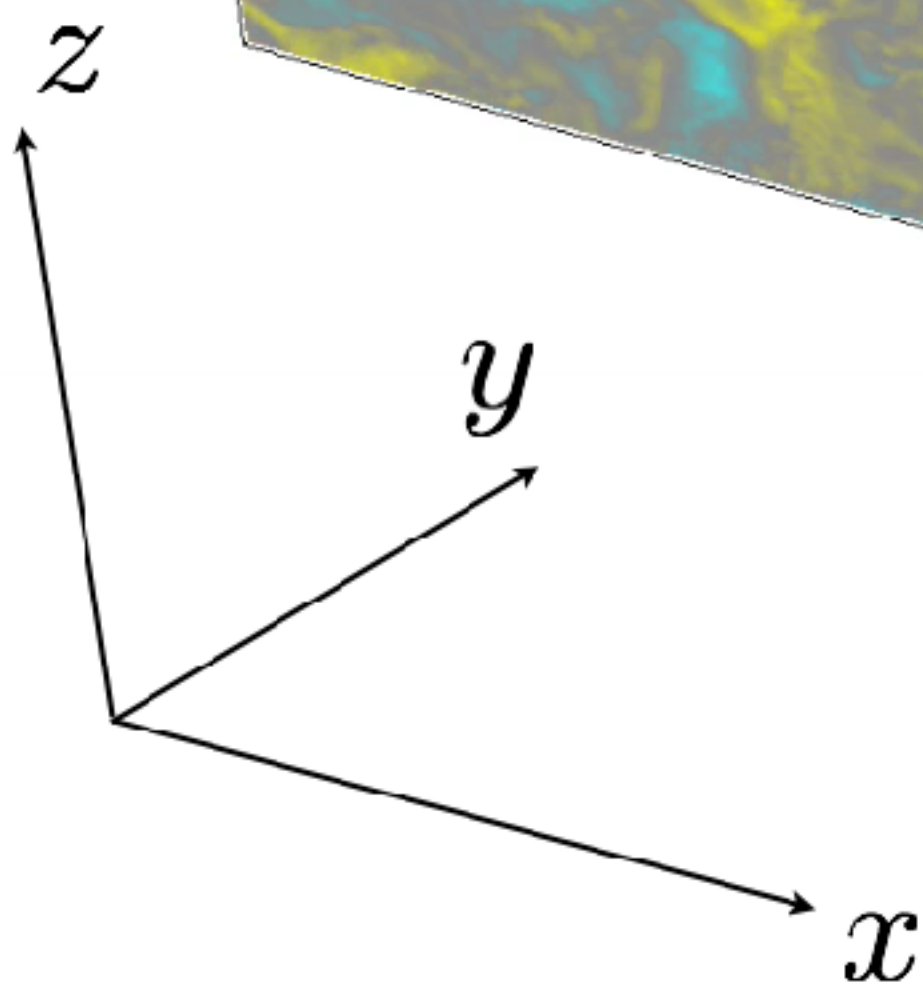
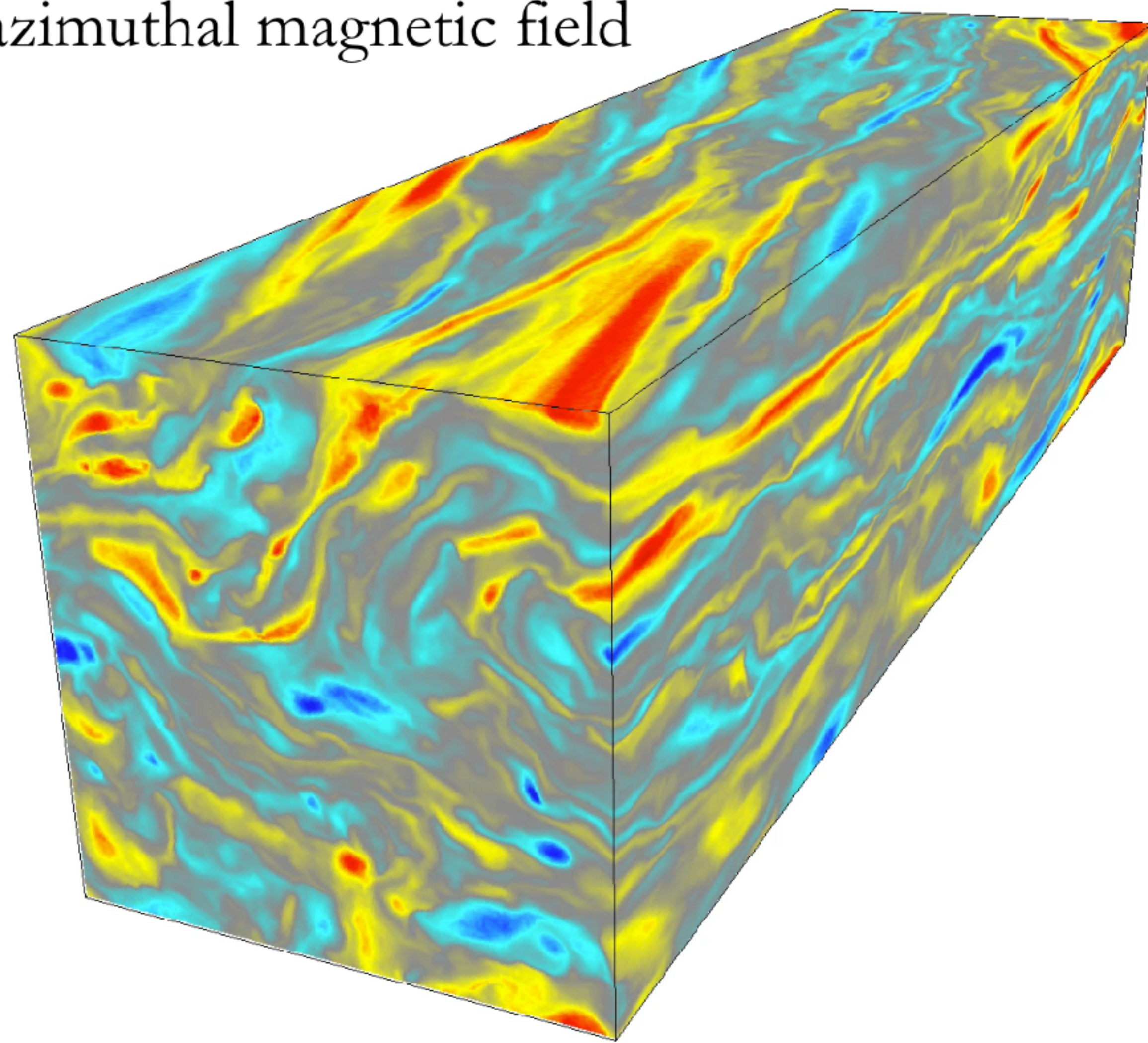
1. What is the nonlinear saturated state of the MRI in a collisionless plasma?
2. How is marginal stability to kinetic instabilities achieved and maintained as the MRI amplifies the magnetic field?
3. What is the effective viscosity (and conductivity) in these disks? What role does collisionless wave damping play in limiting the efficacy of large-wavelength modes to transport angular momentum?
4. Connection between angular-momentum transport and particle heating?
5. How do these properties vary with radius and height in a global setting?



radial magnetic field  
(times 2)



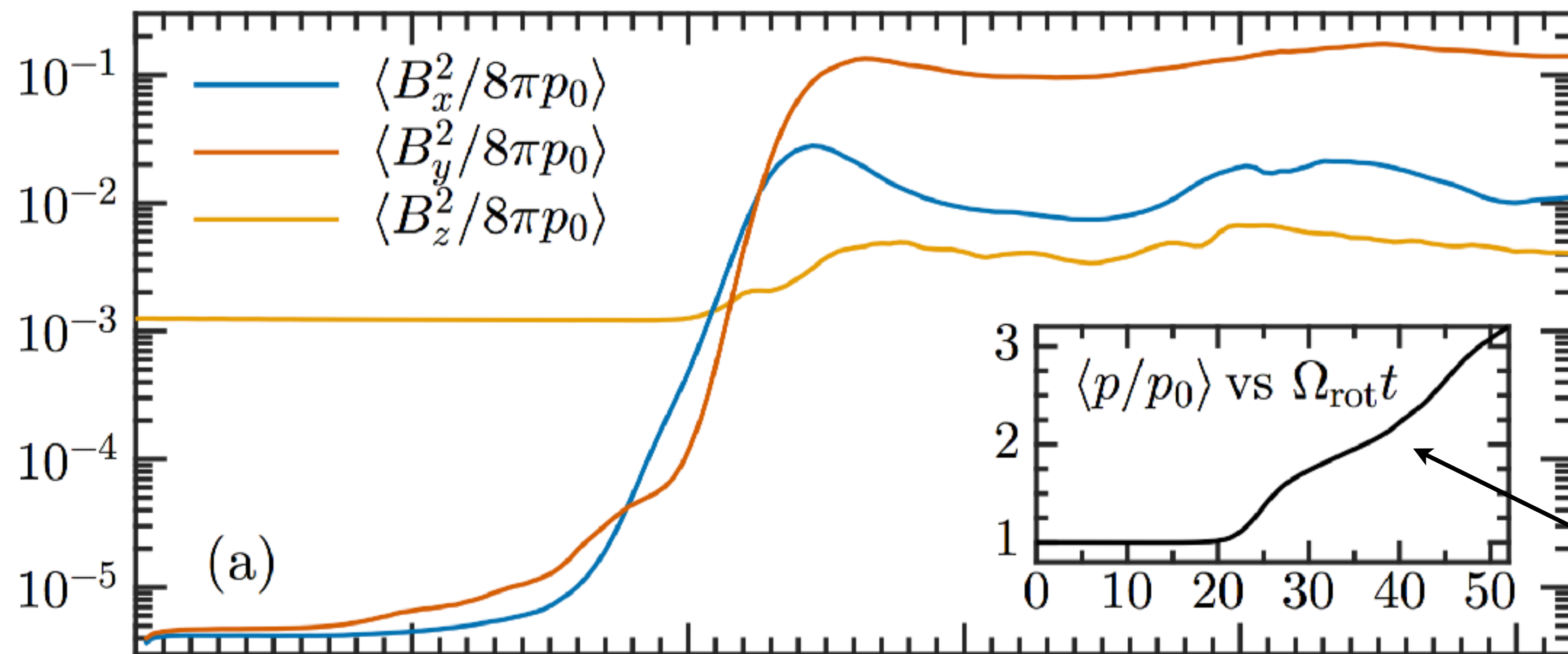
azimuthal magnetic field



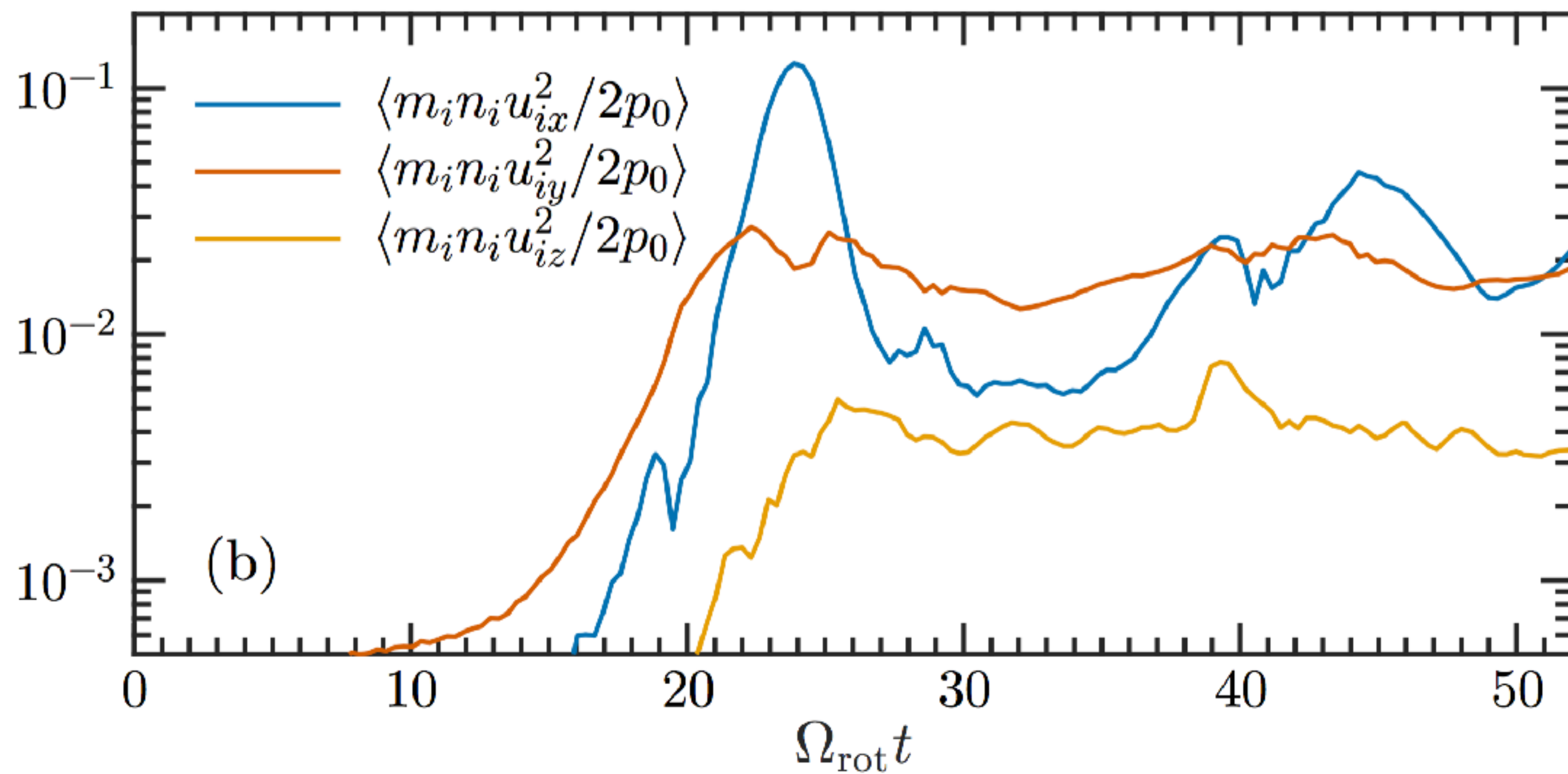


# box-averaged energies vs. time

magnetic energy  
(azimuthal dominates)



kinetic energy

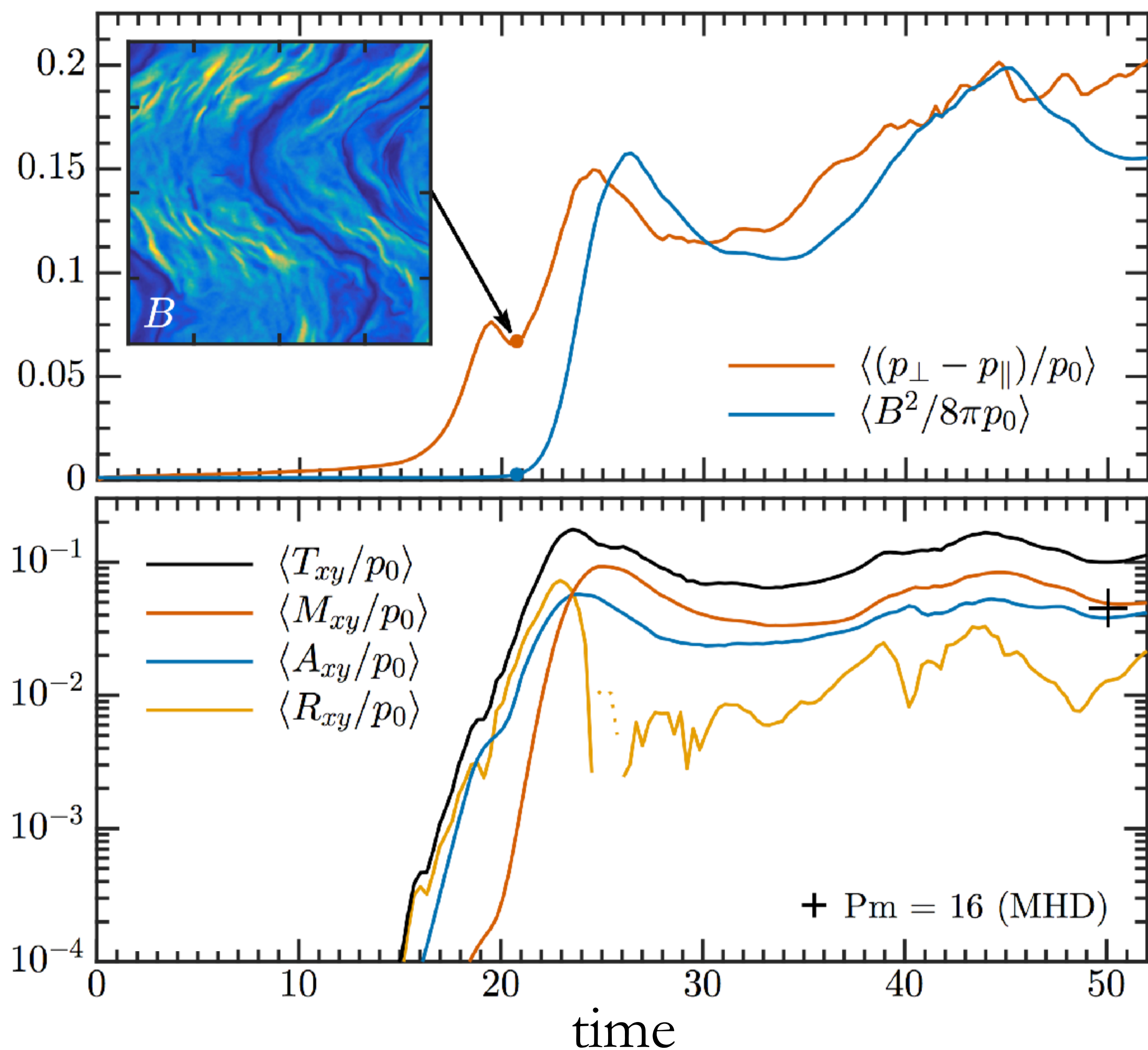




MRI drives **pressure anisotropy**,  
 triggers mirror modes,  
**regulate** pressure anisotropy  
 (time lag related to insufficient  
 scale separation ...more later)

angular-momentum transport  
 due to **pressure anisotropy**  
 is comparable to (usual)  
 transport by **Maxwell stress**

direct connection between  
 plasma micro-physics and  
 macro-scale dynamics

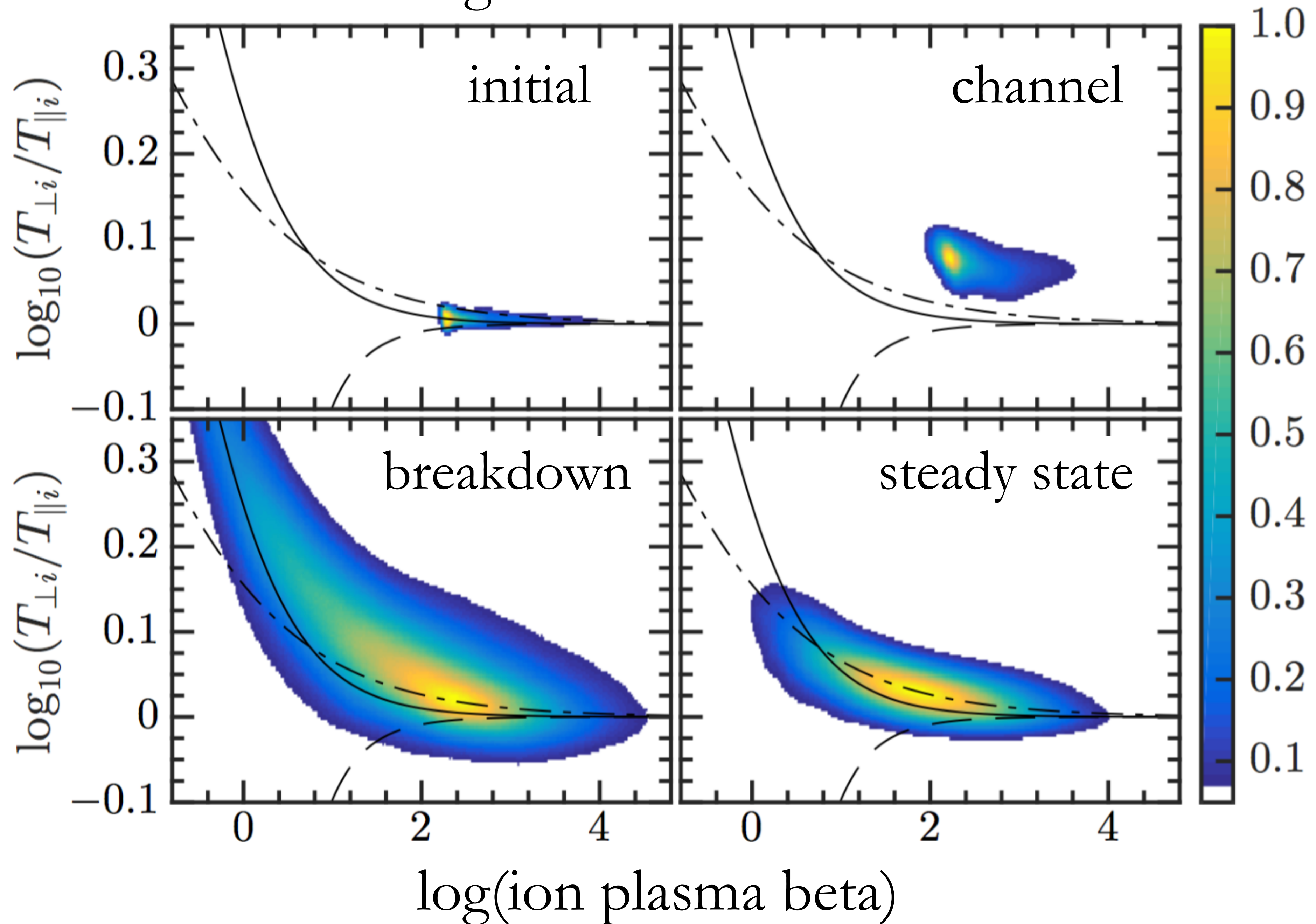




# histogram of simulation data

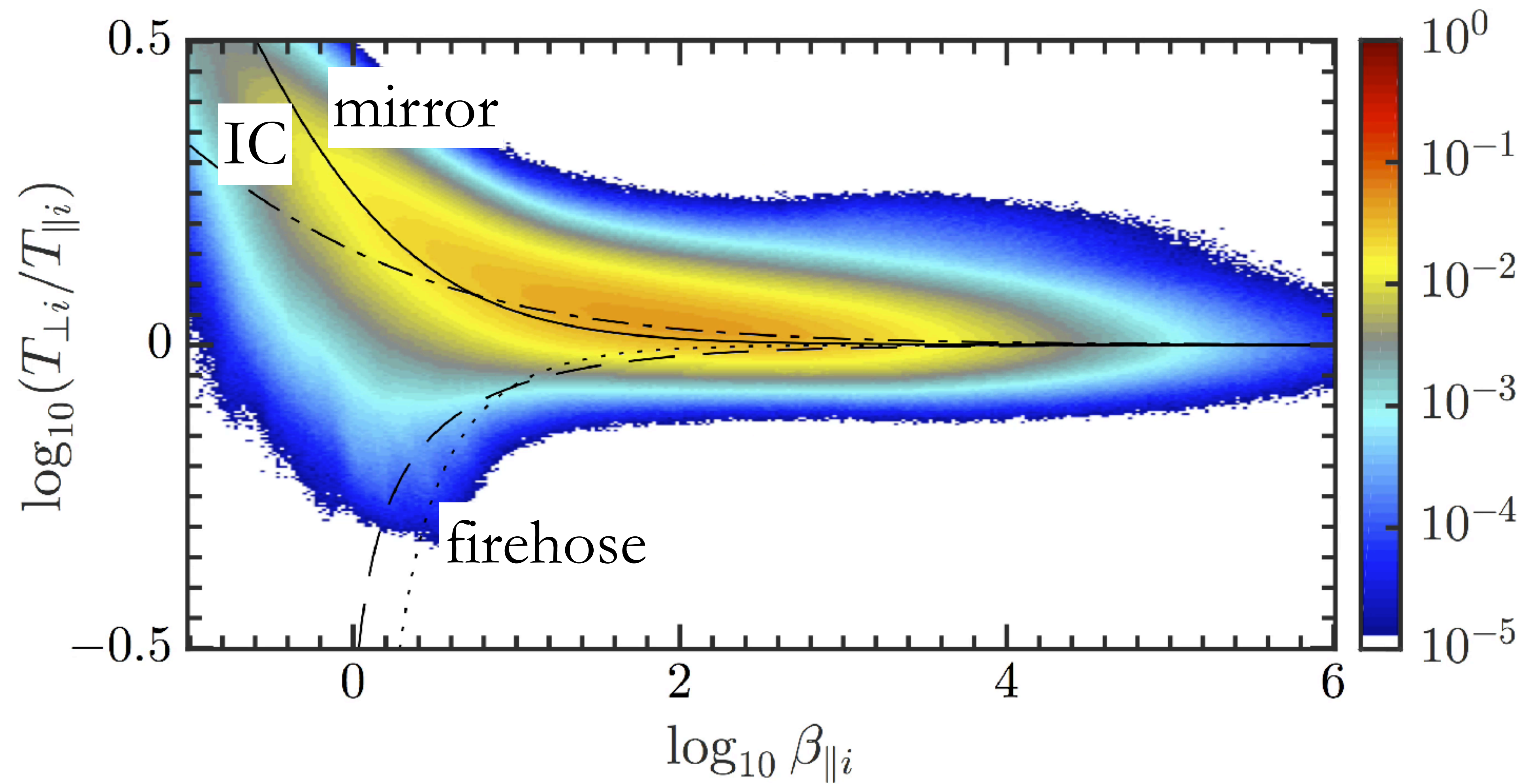
log(ion pressure  
anisotropy)  
(0 = isotropic)

solid line  
is the  
mirror  
instability  
threshold



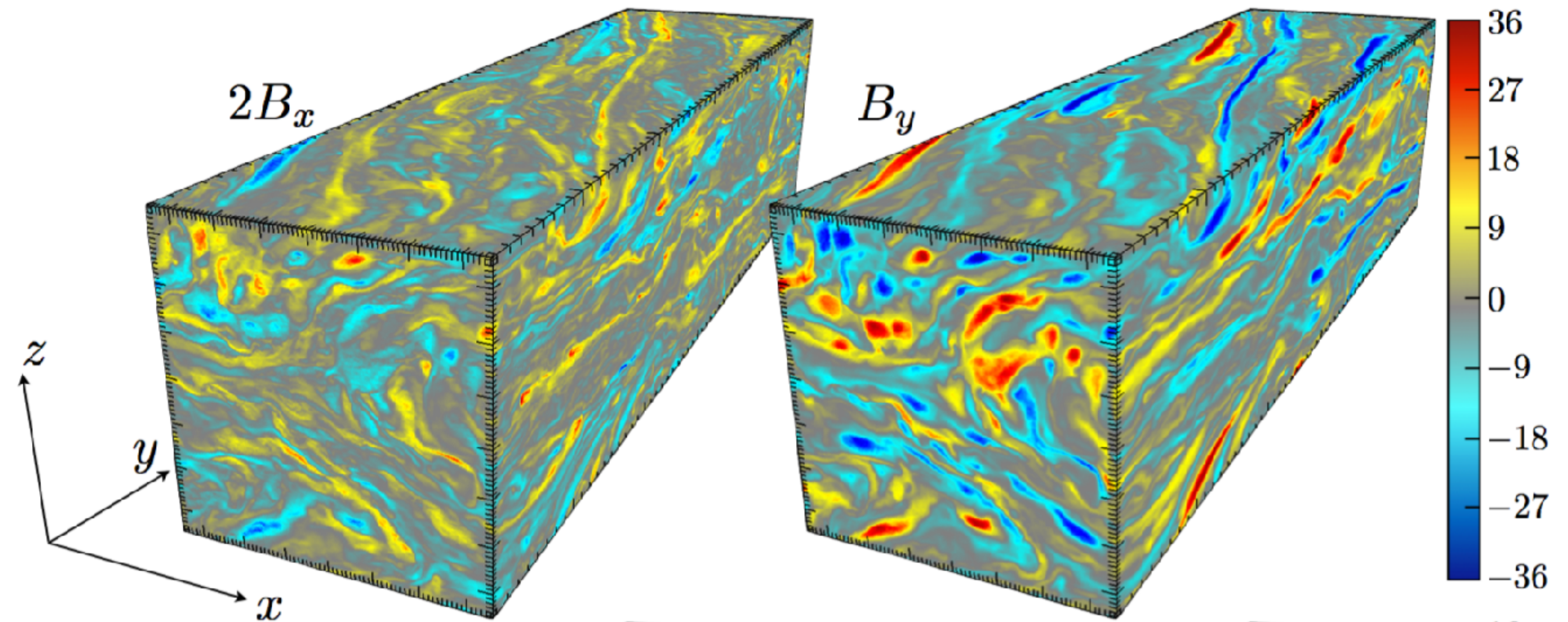


$$\log_{10} N(T_{\perp i}/T_{\parallel i}, \beta_{\parallel i})$$

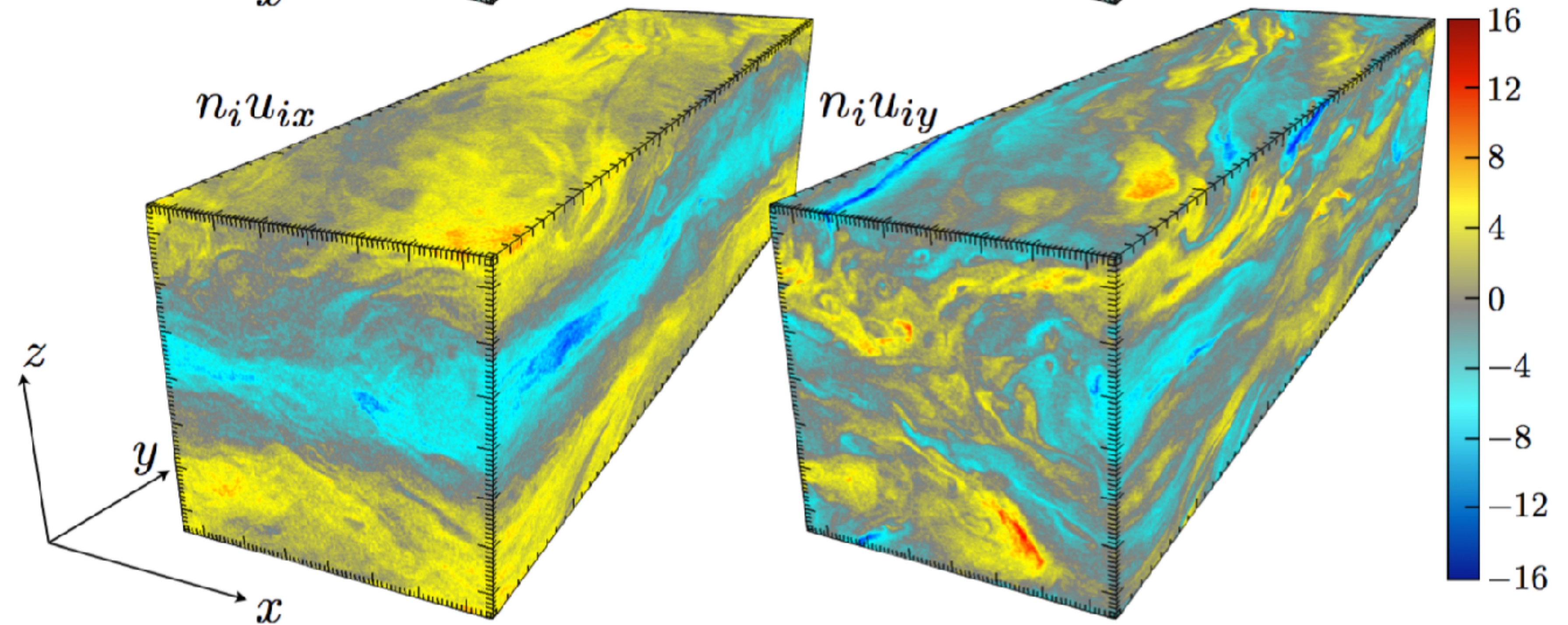




radial and azimuthal  
magnetic-field  
fluctuations



radial and azimuthal  
(ion) momentum  
fluctuations



behaves like a  
 $P_m \gg 1$  plasma



# Energy vs. wavenumber

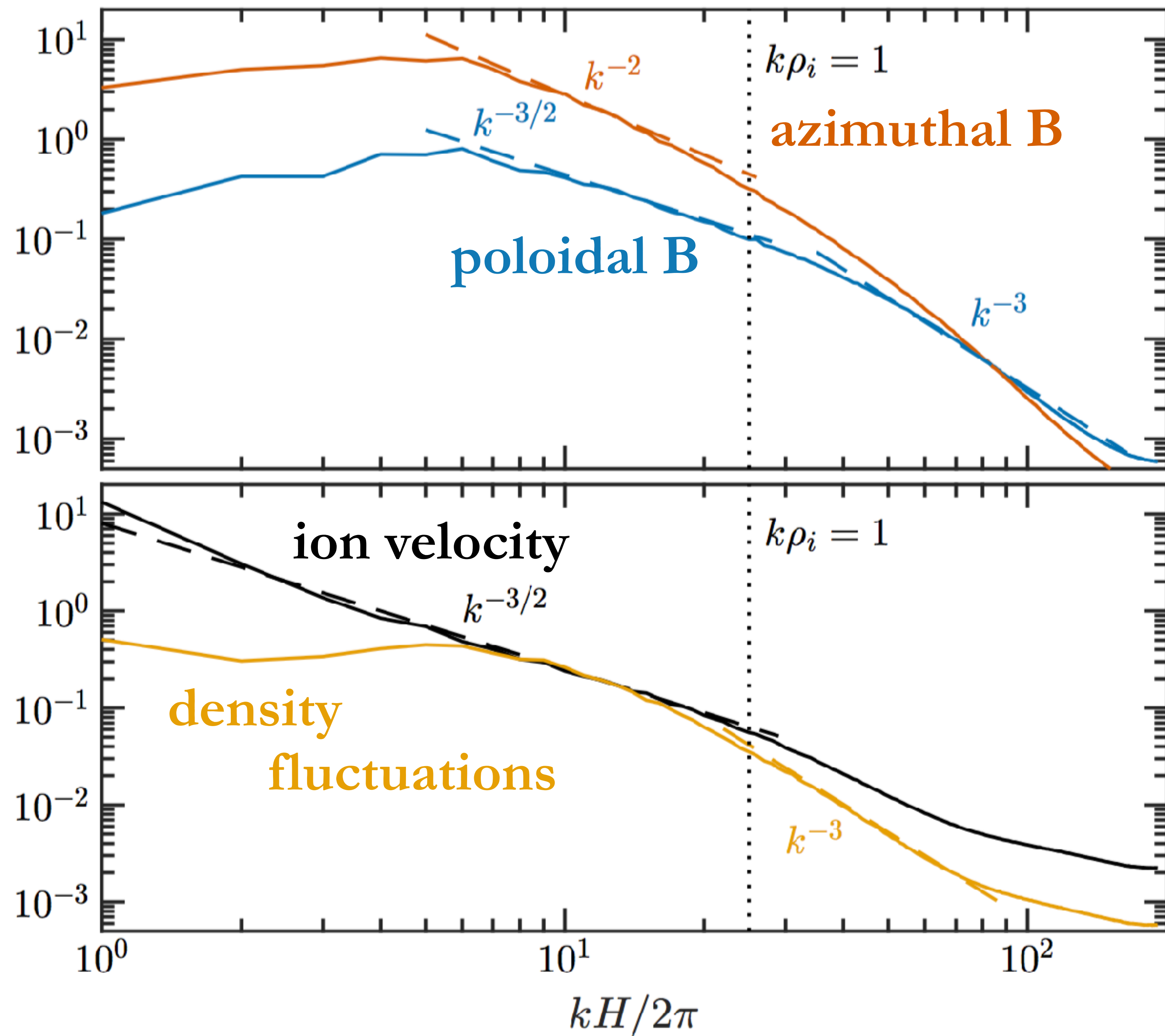
$$k\rho_i < 1$$

similar to MHD studies  
(e.g., Walker, Boldyrev & Lesur 2016)

Alfvén-wave cascade along locally  
azimuthal “guide” field

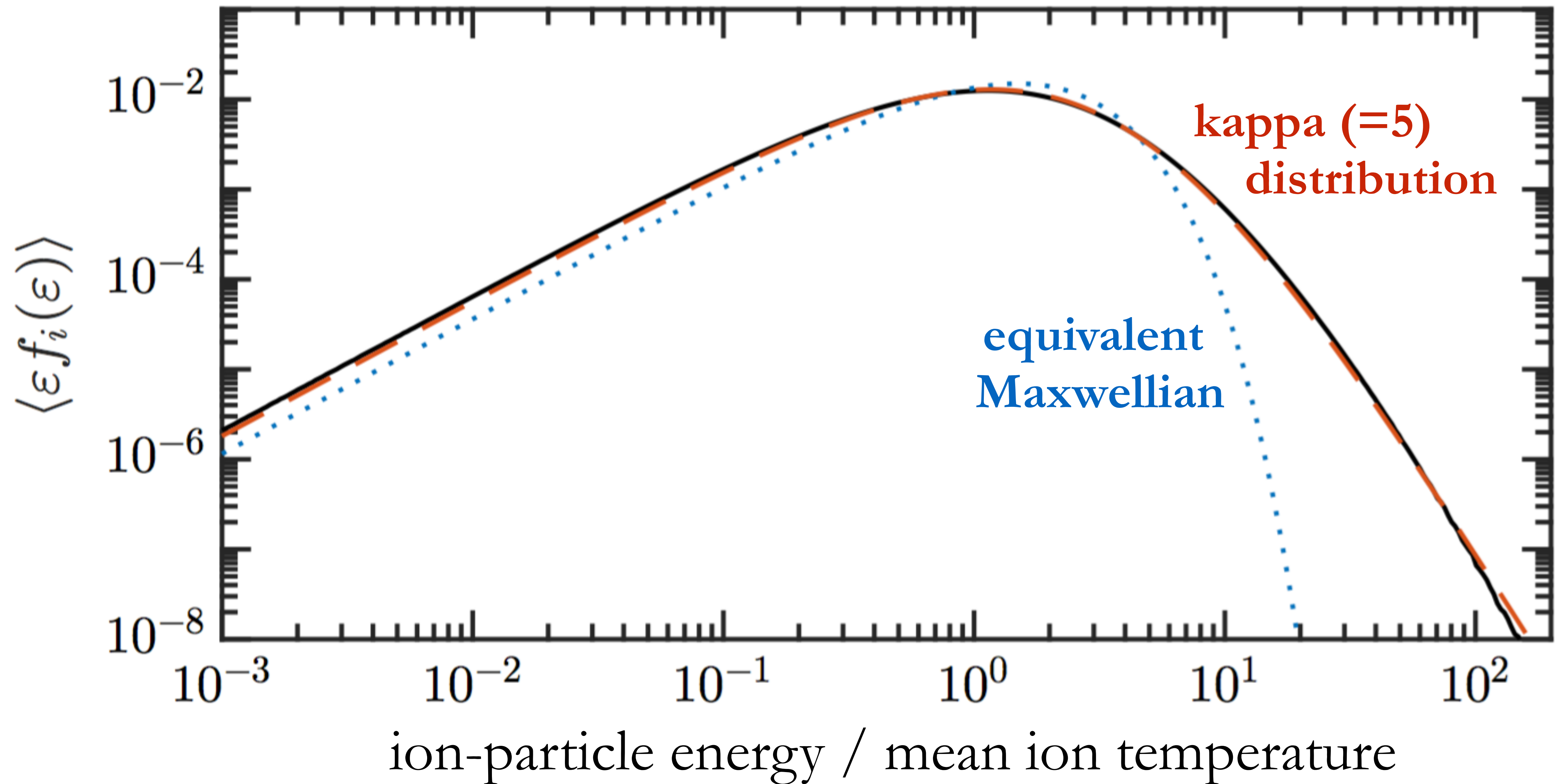
$$k\rho_i > 1$$

kinetic-Alfvén-wave cascade  
(-3, like in solar wind)





# ion-particle energy spectrum at end of run





## Profits, Perils, and Price (of our approach)

Profits: First 3D3V kinetic simulation of magnetorotational turbulence and dynamo — *feasible!*

*Ab initio* demonstration that pressure anisotropy enhances angular-momentum transport in a way controlled by the kinetic microphysics

Despite the absence of (explicit) interparticle collisions, much at large scales looks like (anisotropic,  $P_m \gg 1$ ) MHD

Can afford good scale separation, which can capture both Alfvénic and kinetic-Alfvén-wave cascade



# Profits, Perils, and Price (of our approach)

Perils: Massless, isothermal, fluid electrons (can improve this)

No electron heating included (can improve this)

More scale separation and orbits would be better

Price:  $\text{cost} \propto (k_{\text{max}} \rho_{i0})^4 \left( \frac{\Omega_{i0}}{\Omega_{\text{rot}}} \right)^{-4} \dots \text{ouch}$

*BUT*, doing fully kinetic with

$$m_i/m_e = 25 \quad c/v_{th,e} = 5 \quad \Delta x = \lambda_D/2$$

4 particles per cell and all else the same

$\sim 10^{12}$  CPU-hours per orbit!



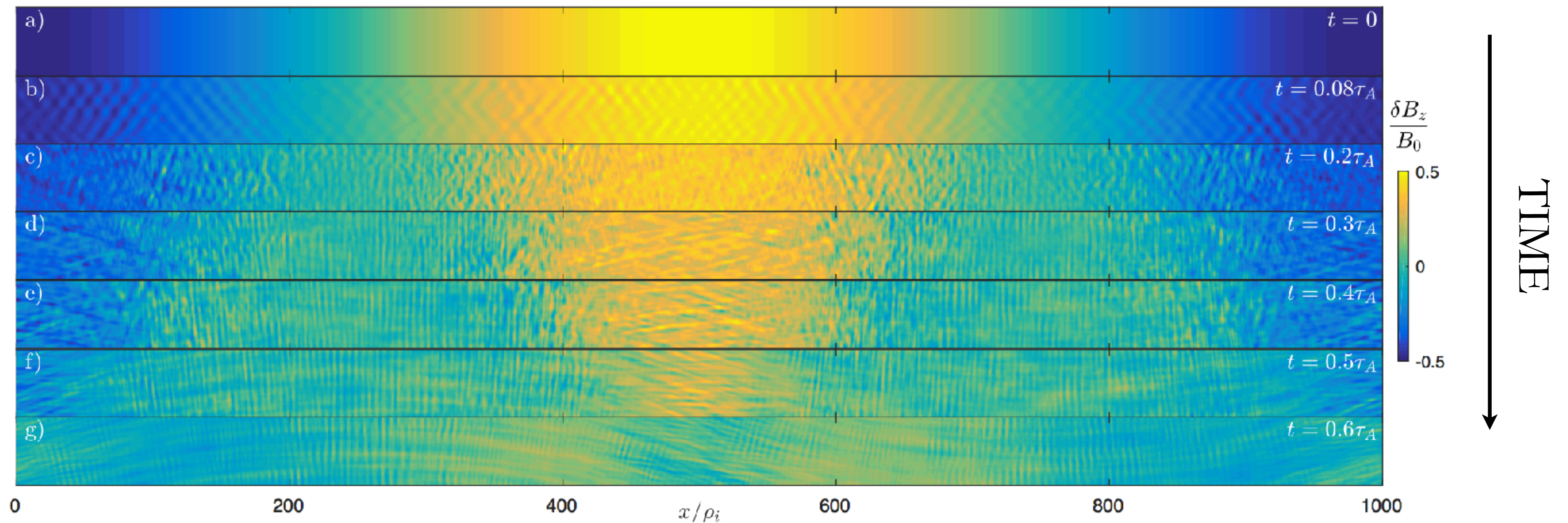
# Conclusions

- (1) Micro-scale physics can play a fundamental role in dictating what macro-scale dynamics are allowed in a given system;
- (2) 3D kinetic simulations of magnetorotational turbulence and plasma dynamo are now within reach (though expensive);
- (3) On the average, the MRI produces positive pressure anisotropy, which is regulated by mirror and ion-cyclotron instabilities ( *ab initio proof!* );
- (4) Collisionless plasma acts like a high-Pm plasma, with a  $\beta$ -dependent effective viscosity. Beautiful energy spectra, with Alfvén-wave and kinetic-AW cascades.
- (5) Despite prior theoretical work and an awareness of the issues involved, I am *still* amazed that a collisionless turbulent plasma self-organizes to produce MHD-like behavior at large scales.



small adverts for some things I'm also excited about...

collisionless plasmas cannot support linearly polarized Alfvén waves above  $\delta B_{\perp}/B_0 \sim \beta^{-1/2}$



Squire, Kunz, Quataert & Schekochihin, PRL, in press

Squire, Quataert & Schekochihin, 2016, ApJ, 830, 25